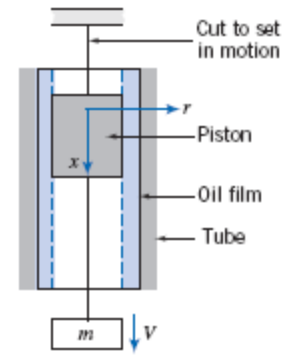


Problem 2.49

[Difficulty: 3]

2.49 The piston in Problem 2.48 is traveling at terminal speed. The mass m now disconnects from the piston. Plot the piston speed vs. time. How long does it take the piston to come within 1 percent of its new terminal speed?



Given: Flow data on apparatus

Find: Sketch of piston speed vs time; the time needed for the piston to reach 99% of its new terminal speed.

Solution:

Given data: $D_{\text{piston}} = 73 \cdot \text{mm}$ $D_{\text{tube}} = 75 \cdot \text{mm}$ $L = 100 \cdot \text{mm}$ $SG_{\text{Al}} = 2.64$ $V_0 = 10.2 \cdot \frac{\text{m}}{\text{s}}$

Reference data: $\rho_{\text{water}} = 1000 \cdot \frac{\text{kg}}{\text{m}^3}$ (maximum density of water) (From Problem 2.48)

From Fig. A.2, the dynamic viscosity of SAE 10W-30 oil at 25°C is: $\mu = 0.13 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$

The free body diagram of the piston after the cord is cut is:

Piston weight: $W_{\text{piston}} = SG_{\text{Al}} \cdot \rho_{\text{water}} \cdot g \cdot \left(\frac{\pi \cdot D_{\text{piston}}^2}{4} \right) \cdot L$

Viscous force: $F_{\text{viscous}}(V) = \tau_{rz} \cdot A$

or $F_{\text{viscous}}(V) = \mu \cdot \left[\frac{V}{\frac{1}{2} \cdot (D_{\text{tube}} - D_{\text{piston}})} \right] \cdot (\pi \cdot D_{\text{piston}} \cdot L)$

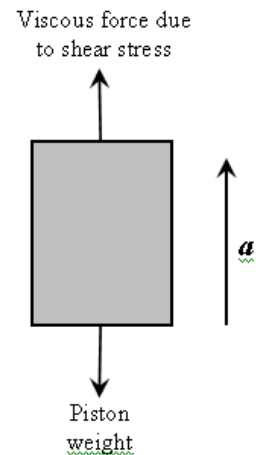
Applying Newton's second law: $m_{\text{piston}} \cdot \frac{dV}{dt} = W_{\text{piston}} - F_{\text{viscous}}(V)$

Therefore $\frac{dV}{dt} = g - a \cdot V$ where $a = \frac{8 \cdot \mu}{SG_{\text{Al}} \cdot \rho_{\text{water}} \cdot D_{\text{piston}} \cdot (D_{\text{tube}} - D_{\text{piston}})}$

If $V = g - a \cdot V$ then $\frac{dX}{dt} = -a \cdot \frac{dV}{dt}$

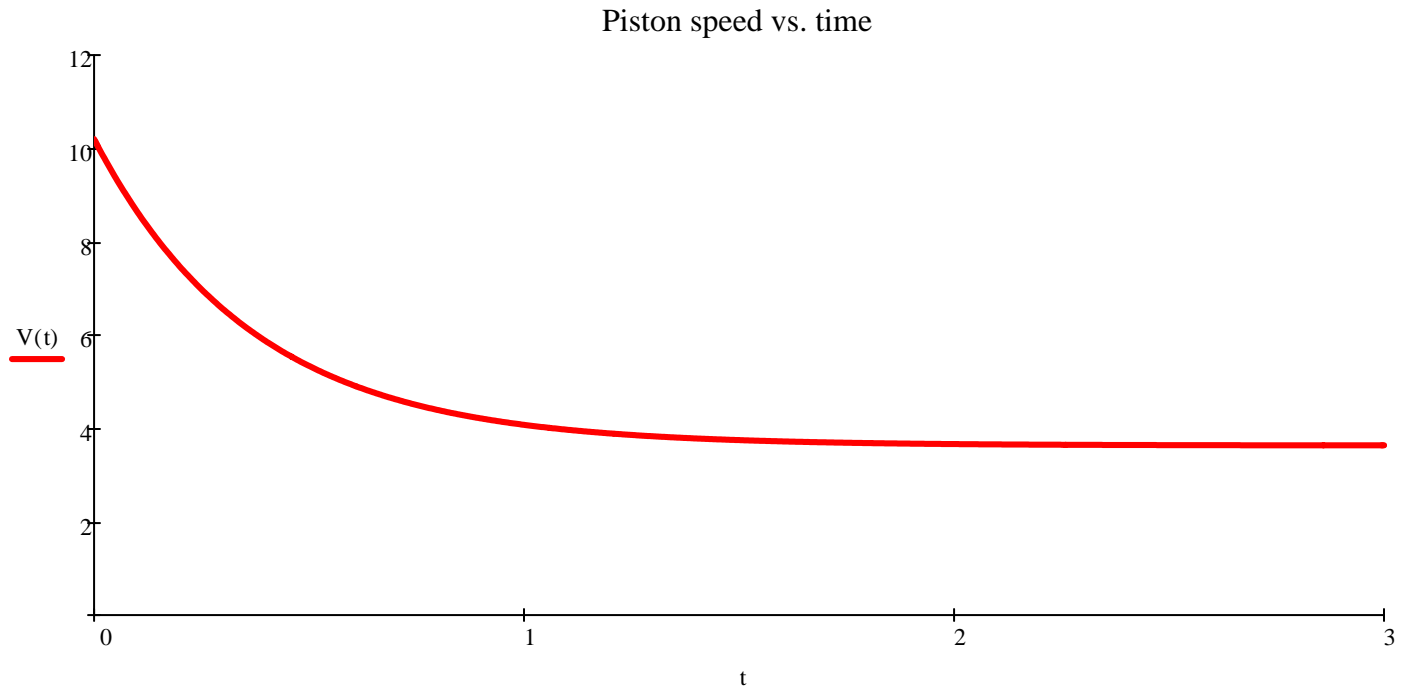
The differential equation becomes $\frac{dX}{dt} = -a \cdot X$ where $X(0) = g - a \cdot V_0$

The solution to this differential equation is: $X(t) = X_0 \cdot e^{-a \cdot t}$ or $g - a \cdot V(t) = (g - a \cdot V_0) \cdot e^{-a \cdot t}$



Therefore
$$V(t) = \left(V_0 - \frac{g}{a} \right) \cdot e^{(-a \cdot t)} + \frac{g}{a}$$

Plotting piston speed vs. time (which can be done in Excel)



The terminal speed of the piston, V_t , is evaluated as t approaches infinity

$$V_t = \frac{g}{a} \quad \text{or} \quad V_t = 3.63 \frac{\text{m}}{\text{s}}$$

The time needed for the piston to slow down to within 1% of its terminal velocity is:

$$t = \frac{1}{a} \cdot \ln \left(\frac{V_0 - \frac{g}{a}}{1.01 \cdot V_t - \frac{g}{a}} \right) \quad \text{or} \quad t = 1.93 \text{ s}$$